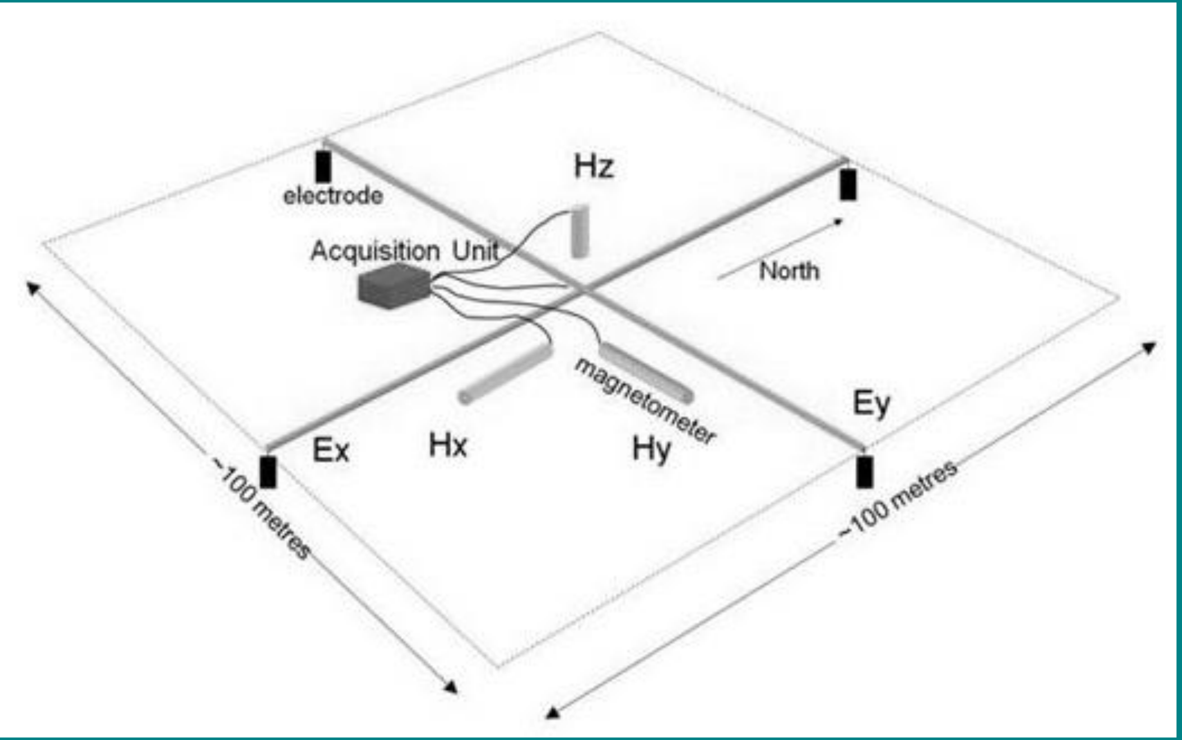
# 06_The MT transfer function

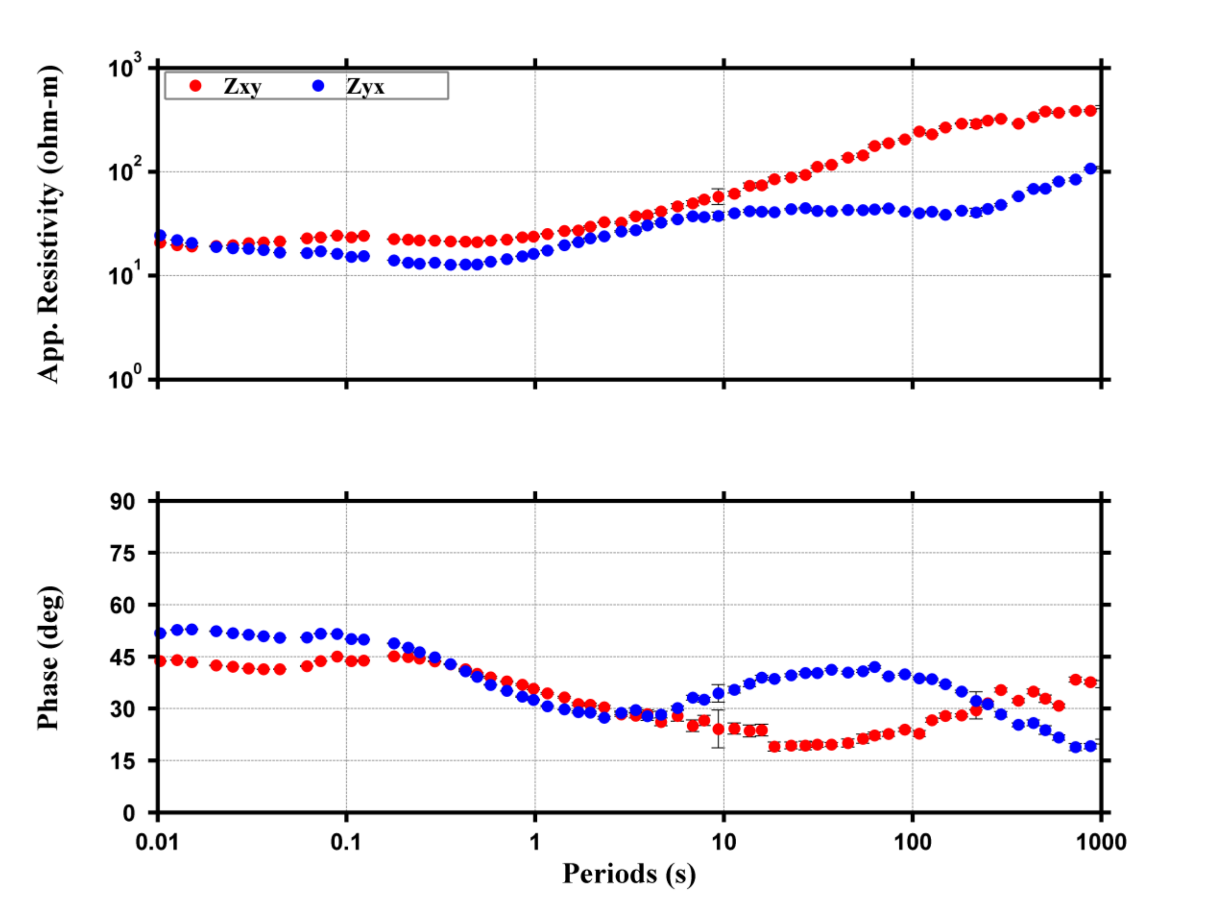
# Magnetotelluric measurement

Measures the time-variations in the horizontal electric fields ($E_x$ and $E_y$) as well as the the horizontal and vertical magnetic fields ($B_x$, $B_y$ and $B_z$).

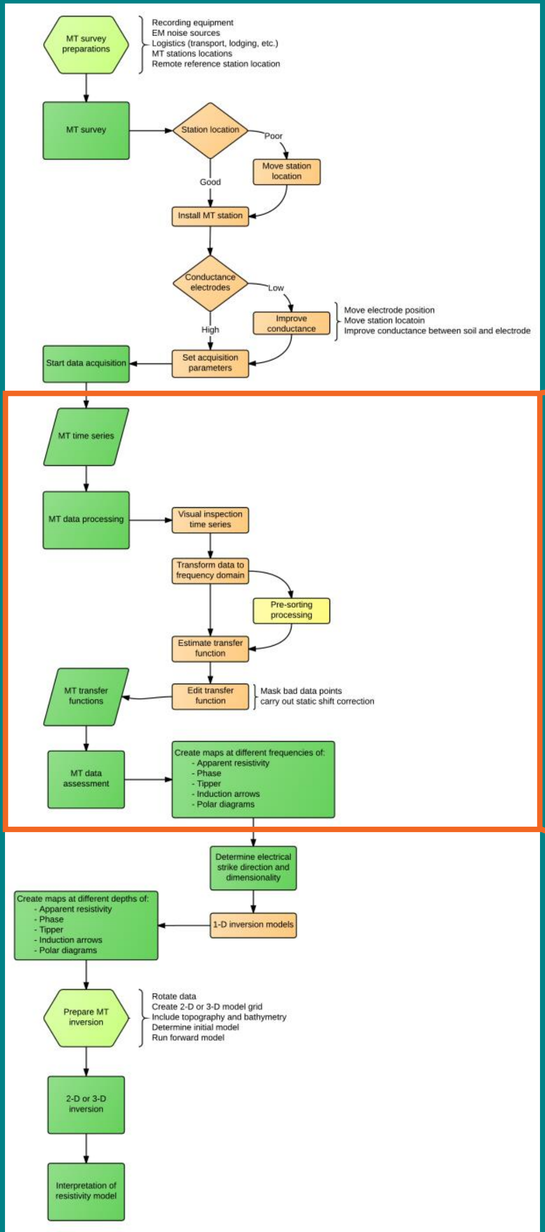
- E-fields are measured with (polarized) electrodes.
- B-fields are measured with magnetic coils.
- The 5 fields are recorded (in time) by a car battery powered data-logger.

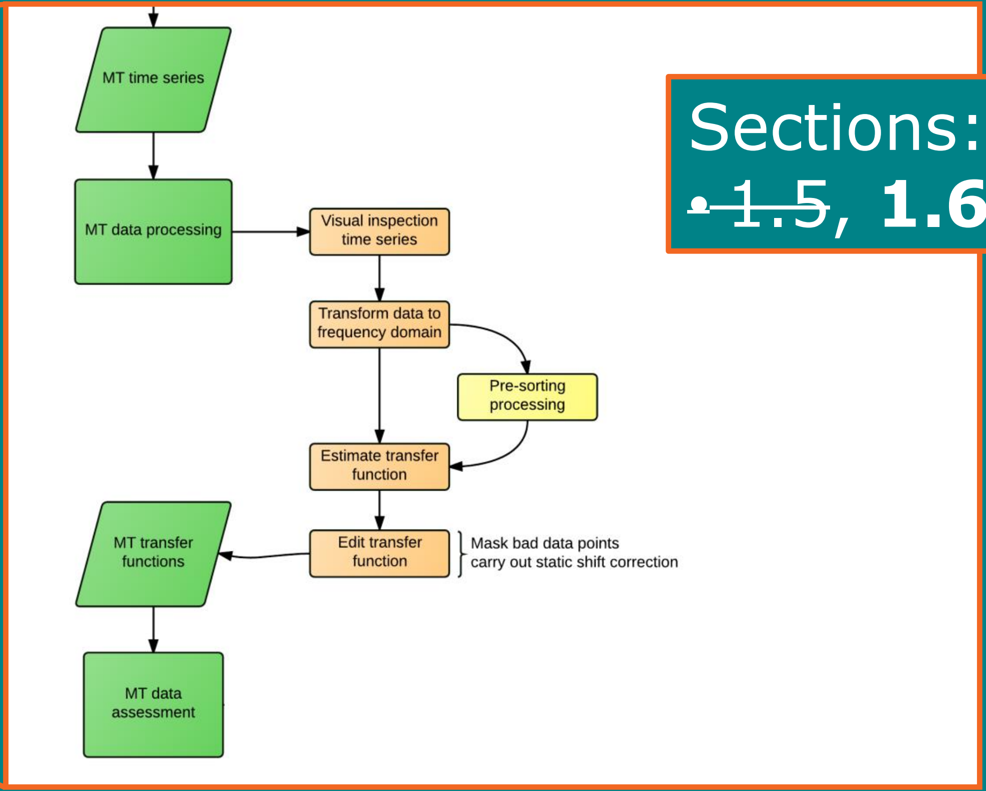


# Magnetotelluric response

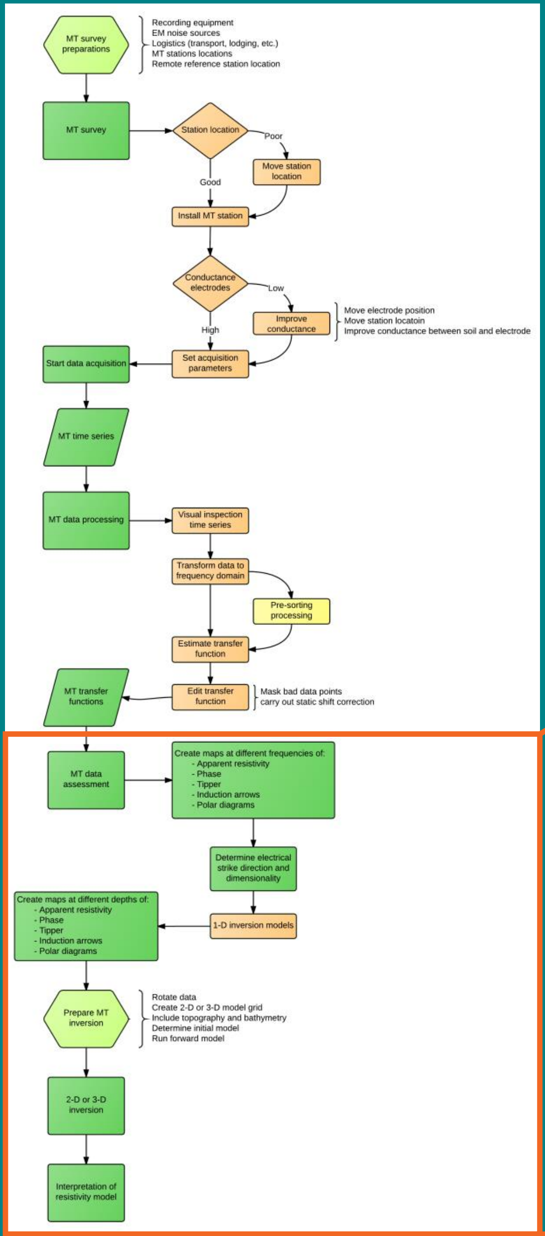


# The MT process (1)

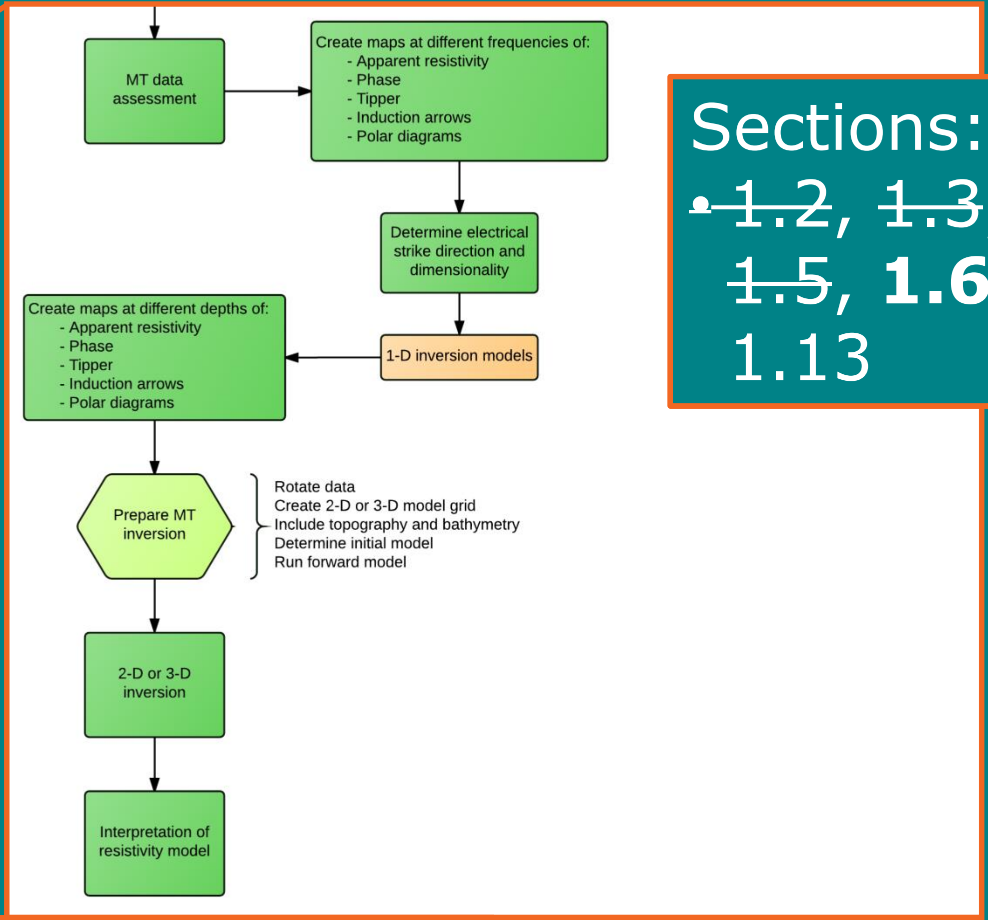



Sections:
- ~~1.5~~, **1.6**, 1.11




# The MT process (2)




Sections:

- ~~1.2~~, ~~1.3~~, ~~1.4~~ ~~1.5~~, **1.6**, 1.12, 1.13

# The magnetotelluric transfer function (Z)

$$\boldsymbol{E}_h = \overline{\boldsymbol{Z}} \cdot \boldsymbol{B}_h$$

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$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \cdot \begin{pmatrix} B_x \\ B_y \end{pmatrix} .$$

- Relates the measured horiztonal electric (E) and horizontal magnetic (B) fields
- Also known as:
  - Impendance tensor
  - MT response function
  - …

# The vertical magnetic transfer function (T)

$$B_z = \boldsymbol{T} \cdot \boldsymbol{B}_h.$$

- Relates the measured vertical and horizontal magnetic (B) fields
- Also known as Tipper

- Derivation of the MT transfer function? Let’s (try to) keep it simple.
  - For a layered Earth

$$\nabla^2 \boldsymbol{E} = \mu_0 \sigma \frac{\partial \boldsymbol{E}}{\partial t} = i\omega\mu_0\sigma\boldsymbol{E}.$$

  - $\mu_0$ = free-space magnetic permeability
  - $\sigma$ = conductivity ($1/\rho$)

- Assume an insulating uniform half-space at z = 0

$$\frac{\partial E_x}{\partial z} = -kE_x = -\frac{\partial B_y}{\partial t} = -i\omega B_y$$

- $k^2 = i\omega\mu_0\sigma$
- Now the horizontal electric and magnetic fields are linearly related.

$$Z_{xy} = \frac{E_x}{B_y}.$$

Fo(u)r all components:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \cdot \begin{pmatrix} B_x \\ B_y \end{pmatrix}.$$

# Apparent resistivity and phase (1)

- The apparent resistivity, as a function of frequency, can now be computed using the Smucker-Weidelt C-response and assuming a 2-D Earth ($Z_{xx} = Z_{yy} = 0$).

$$C = \frac{1}{k} = \frac{E_x}{i\omega B_y} = -\frac{E_y}{i\omega B_x}$$

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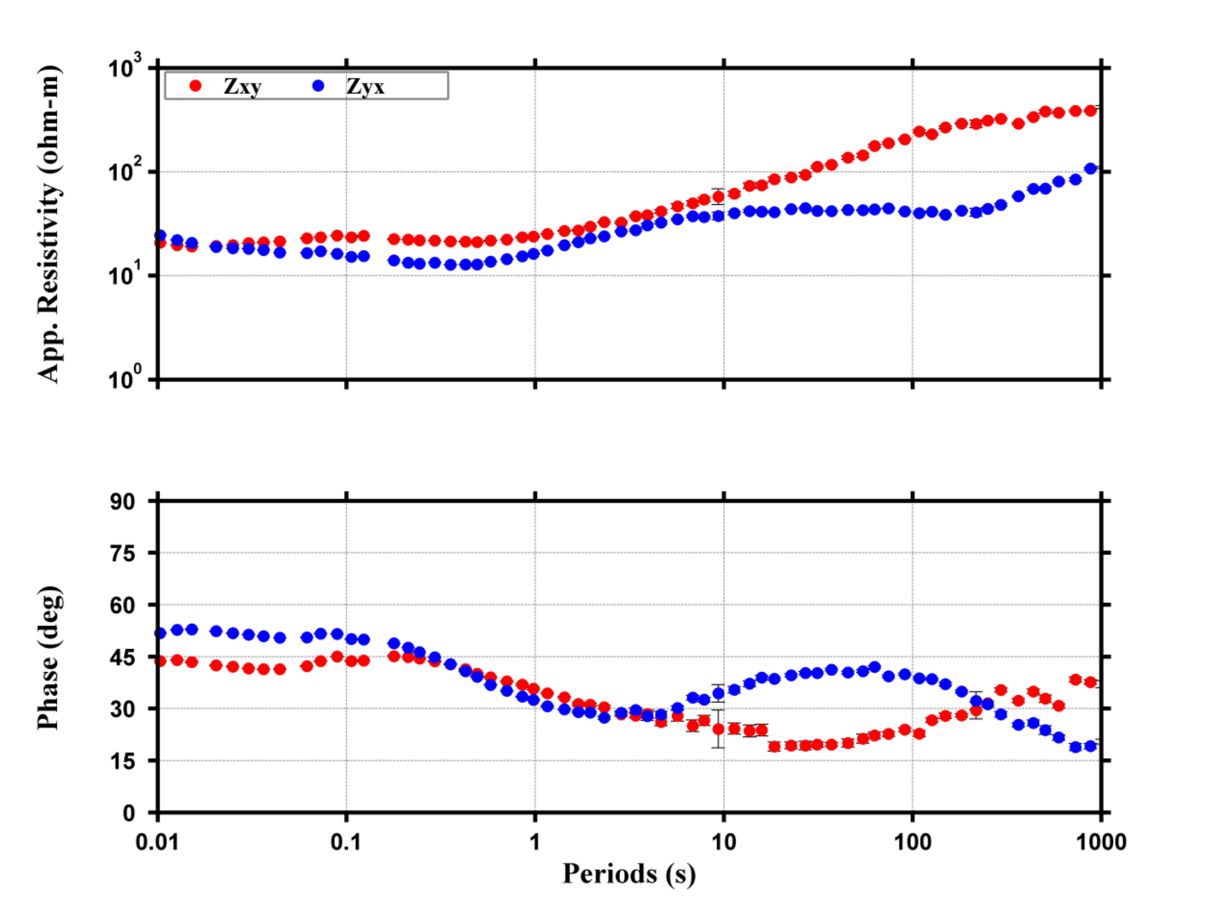
$$\rho_a = \frac{1}{\sigma} = \mu_0 \omega |C|^2.$$

- The phase, as a function of frequency, is computed directely from ***Z***

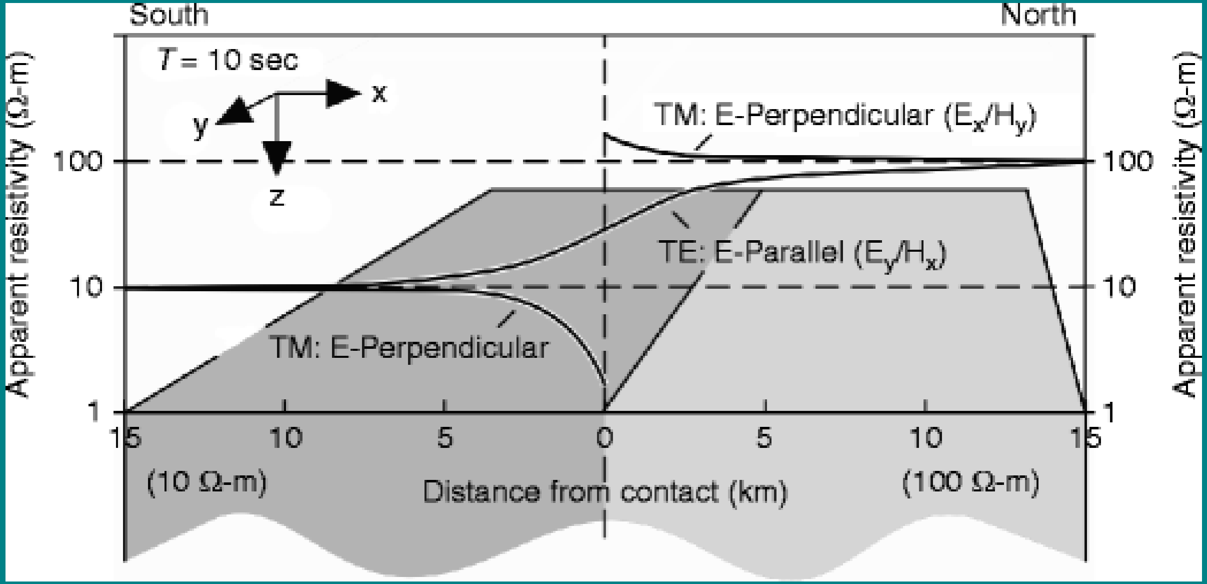
$$\phi_{xy} = \arctan\left[\frac{im(Z_{xy})}{re(Z_{xy})}\right]$$

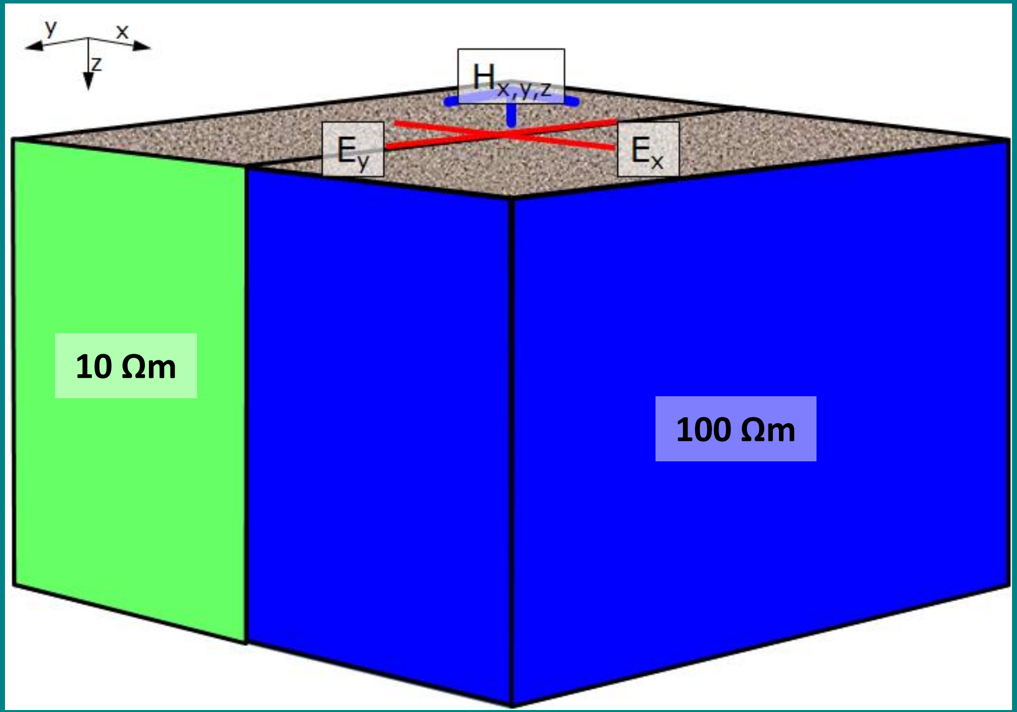
# Apparent resistivity and phase (2)



# Properties MT transfer function (1)

- Polarization:
  - TM-mode (or B-polarization)
  - TE-mode (or E-polarization)



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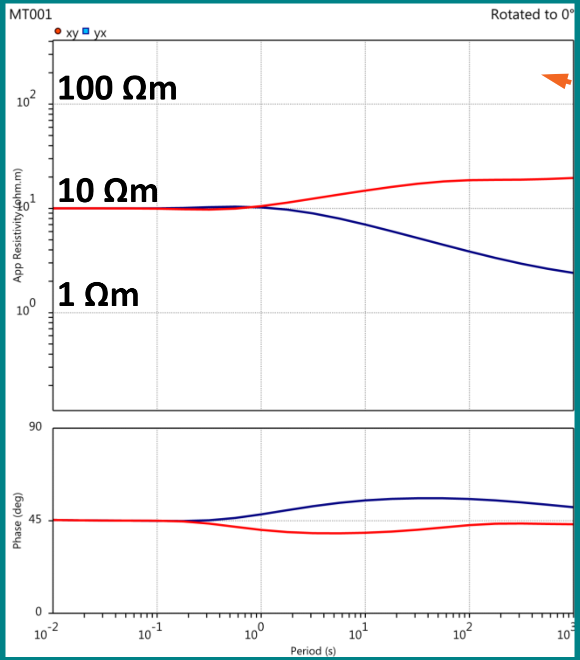
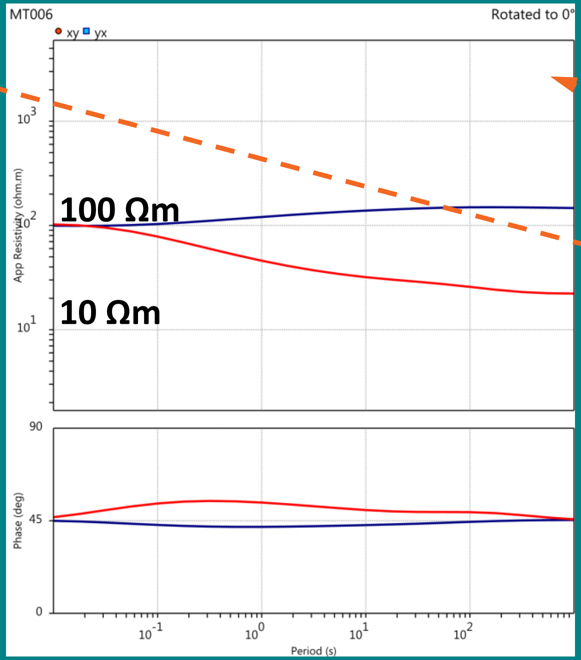
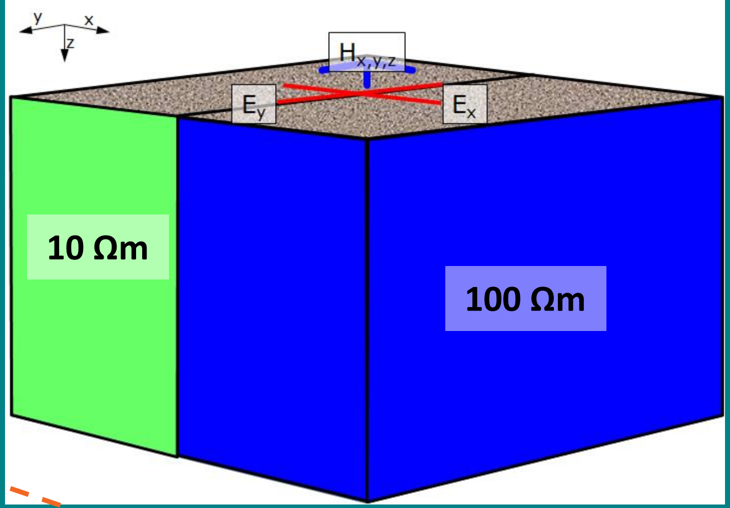
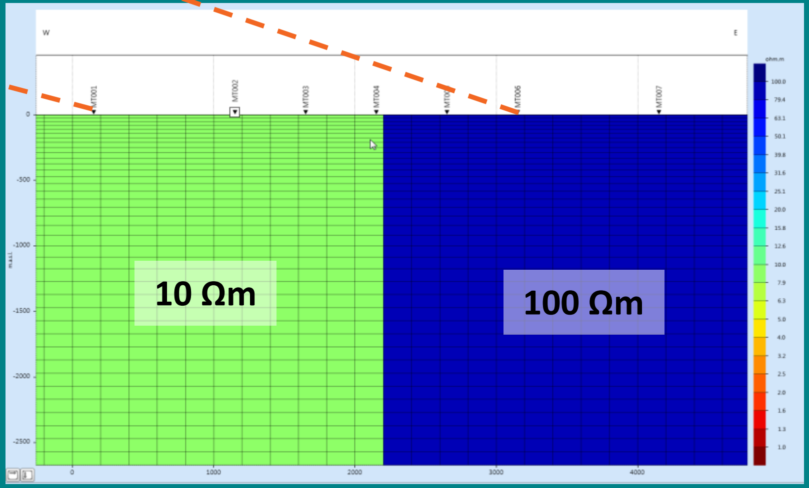


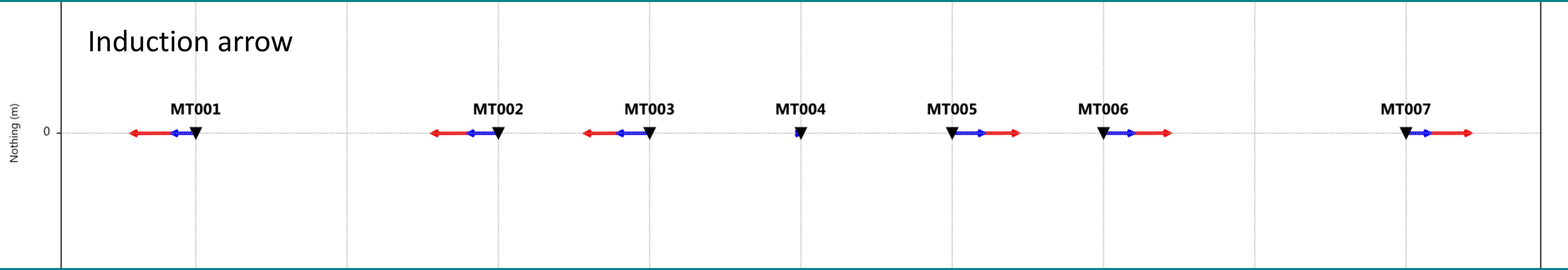



# Properties MT transfer function (2)

- Rotational invariants
  - Properties that hold for any orientation of the horizontal coordinate system. Used as e.g.:
    - dimensionality indicators,
    - first guess resistivity structure
- Induction arrows



# Properties MT transfer function (3)

- Polar diagrams with Tipper strike